The Quantum Adiabatic Approximation in Chaotic Systems

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Abstract

Quantum systems with chaotic classical counterparts cannot be treated by perturbative techniques or any kind of adiabatic approximations. This is so, in spite of the quantum suppression of classical chaos. We explicitly calculate the adiabaticity parameter for the case of a Fermi Accelerator and show that the adiabatic expansion of the evolution operator is divergent. The relevance of this situation for the understanding of quantal effects in the one body dissipation mechanism in nuclear reactions is briefly discussed.

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Over the last years, considerable effort has been directed towards the study of the quantum evolution of simple (few degrees of freedom) classically chaotic systems. Examples of these kind of systems are the well-known Kicked Rotor [1] and the Fermi Accelerator [2]-[5]. This last system is of particular interest since, at the classical level, it is a model for the One Body Dissipation mechanism [6]. Its relevance for the description of nuclear reactions at intermediate energies is now being investigated, as new knowledge on the quantum suppression of classical chaos becomes available [7]. There is increasing evidence [8] that if quantum effects are taken into account, they will tend to inhibit this mechanism. The adiabatic approximation has been commonly used in nuclear physics whenever different time scales are present. The issue of wether the use of this approximation in nuclei is actually justified has been previously examined by Nazarewicz [9]. We address this question in the light of recent developments in the understanding of the quantum description of classically chaotic systems. In spite of the quantum suppression of classical chaos, some characteristics of chaotic systems are still present at the quantum level [10]. It is therefore natural to ask wether the adiabatic approximation can be used successfully for quantum systems that are classically chaotic. It is well established, since the pioneering work by H. Poincaré [11] at the end of the last century, that the adiabatic approximation does not hold in the case of classical chaotic systems because the perturbative adiabatic series expansions diverge. In this letter, it is argued that perturbative adiabatic approaches are also divergent for quantum systems with chaotic classical analogs. We explicitly work out the case of a quantum Fermi Accelerator model and discuss other similar examples which cannot be treated by adiabatic techniques. Finally, the relevance of this situation for the understanding of quantum mechanical effects in the one body dissipation mechanism in nuclear reactions is briefly discussed.

The Fermi Accelerator [12] consists of a particle confined in an infinite well with one periodically moving wall. The motion of the wall can be parametrized as $L(t) = L_0\,[1 + \delta f(t)]$, where δ is the dimensionless amplitude of the wall motion and $f(t)$ is a periodic function scaled so that $|f(t)| \leq 1$. The adiabatic approximation corresponds to the limit $\delta \rightarrow 0$. In the position representation the Hamiltonian operator is $\mathbf{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$, with time dependent boundary conditions imposed on the wavefunction $\Psi(x = 0, t) = \Psi(x = L(t), t) = 0$. No exact solution is known for this problem unless $\delta = 0$. Following Makowski [3], we use the transformation $y = \frac{x}{L}$

and after an adequate redefinition of the phase of the wavefunction, the problem can be stated in terms of time independent boundary conditions $\varphi(y=0,t)=\varphi(y=1,t)=0$. This can be done at the cost of introducing a time dependent potential in the transformed Hamiltonian operator

$$\widetilde{\mathbf{H}}=-\frac{\hbar^2}{2mL^2}\frac{\partial^2}{\partial y^2}+\frac{1}{2}mL\frac{d^2L}{dt^2}y^2. \tag{1}$$

For the cases of a sawtooth and triangular wave wall motions, this Hamiltonian has recently been shown to be equivalent to a generalized Kicked Rotor Hamiltonian [13]. In these cases, the function $L(t)$ is not analytic. The evolution operator of impulsive systems, such as the Kicked Rotor, can be obtained explicitly and a quantum map which describes the time evolution of the wavefunction can be constructed [14]. On the other hand, as stated above, in the case of the Fermi Accelerator with a periodically moving wall described by an analytic function of time, the evolution operator has not been obtained in closed form. In fact, the task of obtaining a closed expression for the evolution operator for the important case of sinusoidal wall motion has proved to be a very elusive one [3],[4].

The fact that the effective Hamiltonian for this problem, $\widetilde{\mathbf{H}}$, can be written (Eq. 1) as the sum of an integrable term plus a time dependent potential which is proportional to the perturbation strength δ, has motivated certain authors[15],[16] to suggest the use of perturbative techniques to find an approximate solution for the Schrödinger equation valid in the adiabatic limit $(\delta\rightarrow 0)$. The classical Fermi Accelerator is strongly chaotic in the low velocity limit [12] and thus it is an example of a non-adiabatic system. This is a consequence of the topological changes that the phase space suffers in going from an integrable problem $(\delta=0)$ to a chaotic problem $(\delta\neq 0)$. At the quantum level, the quantum suppression of chaos leaves the question open as wether the adiabatic approximation can be used or not. In the Kicked Rotor, the quantum suppression of classical chaos has been well understood [17] in terms of a mathematical analogy with Anderson localization in solid state physics [18]. Recent numerical results [19] show that a similar suppression of classical chaos is present in the quantum Fermi Accelerator with sinusoidal wall motion and suggest that strong similarities exist between both systems as other authors [20],[13] have pointed out previously. Therefore, the question of wether the adiabatic approximation can be used in the quantum version of these systems, in spite of their classically chaotic nature, does not have an obvious answer.

The general formalism for an adiabatic expansion of the evolution operator of a time-dependent quantum Hamiltonian has been developed recently by A. Mostafazadeh[21]. There, a series expansion of the evolution operator in orders of ν, the adiabaticity parameter defined below, is presented

$$\mathbf{U}(t) = \mathbf{U}^{(0)}(t)\left[\mathbf{T}e^{-\frac{i}{\hbar}\int_0^t ds\mathbf{H}'(s)}\right] = \sum_{p=0}^{N-1}\mathbf{U}^{(p)}(t) + \mathcal{O}(\nu^N) \tag{2}$$

where $\mathbf{U}^{(0)}$ is the evolution operator in the absence of a perturbation ($\delta = 0$), $\mathbf{T}$ is the time ordering operator and $\mathbf{H}'(t)$ is a transformed Hamiltonian defined in [21]. The matrix elements of $\mathbf{H}'$ are proportional (in absolute value) to those of the matrix $\mathbf{A}$ defined below. We show that this kind of expansion of the evolution operator does not converge for the Fermi Accelerator problem with a periodic wall motion. This is so, because the adiabaticity parameter ν is of the order of the dimension of the Hilbert space of the problem and is therefore unbounded.

Let us calculate naively the adiabaticity parameter defined in [21] for the Hamiltonian $\mathbf{H}$ of the Fermi Accelerator problem defined above. We introduce the dimensionless time $\tau = \omega t = \frac{2\pi t}{T}$, in terms of the period of the forcing function $f(t)$. The instantaneous eigenstates satisfy $\mathbf{H}|n,\tau>= \varepsilon_n(\tau)|\,n,\tau>$ with $\varepsilon_n(\tau) = \varepsilon_0\frac{L_0^2}{L^2(\tau)}$. The adiabaticity parameter is defined as $\nu \equiv \frac{1}{\varepsilon_0}Sup\left[\mathbf{A}_{mn}\right]$, where $\varepsilon_0 \equiv \frac{\hbar\pi^2}{2m\omega L_0^2}$ is a convenient energy scale and the matrix $\mathbf{A}_{mn}(\tau)$, related to the rate of change of the instantaneous eigenstates, is defined (for $m \neq n$) as

$$\mathbf{A}_{mn} \equiv < m,\tau\,|\frac{d}{d\tau}|\,n,\tau > . \tag{3}$$

We denote by $Sup[\mathbf{A}_{mn}]$ the minimum upper bound of the elements of the matrix $\mathbf{A}$. In the position representation, the instantaneous eigenstates are $< x,\tau\,|\,n,\tau >= \sqrt{\frac{2}{L(\tau)}}\sin\left(\frac{n\pi x}{L(\tau)}\right)$. The explicit expression for the adiabaticity parameter is

$$\nu = \frac{2}{\varepsilon_0}Sup\left[\frac{1}{L}\left|\frac{dL}{d\tau}\frac{mn}{m^2-n^2}\right|\right]. \tag{4}$$

It is clear from the above equation that for $L(\tau)$ periodic and $\delta \neq 0$, the adiabaticity parameter diverges as $m,n \to \infty$. The series expansion for $\mathbf{U}$ (Eq. 2) does not converge in this case and the divergence cannot be resolved by a redefinition of the adiabaticity parameter [21]. A similar analysis shows that

the same is true for impulsive systems such as the Kicked Rotor, the Quantum Bouncer [5] and other related quantum systems with classical chaotic behavior.

We conclude that periodically driven quantum systems with chaotic classical analogs, cannot be properly treated by perturbative techniques or adiabatic approximations since in these cases, the adiabaticity parameter is unbounded and the series expansion of $\mathbf{U}(t)$ is divergent. Thus, in spite of the quantum suppression of classical chaos, the non-adiabatic nature of classically chaotic systems subsists at the quantum level. In contrast, there are related problems which are classically integrable and for which the adiabatic approximation has been successfully applied [22],[15]. In these cases, the moving wall has a final constant velocity. As a result, the adiabaticity parameter satisfies $\nu < 1$ and the adiabatic expansion of $\mathbf{U}$ is convergent. However, integrable Hamiltonians cannot describe dissipative mechanisms. In particular, in order to assess the importance of quantum effects on the one body dissipation mechanism in nuclei, classically chaotic models must be studied from a quantum mechanical perspective. The transition from the integrable case ($\delta = 0$) and the chaotic case ($\delta \neq 0$) introduces a qualitative change in the nature of the problem, which makes it essentially non-adiabatic and as a consequence it cannot be treated successfully by any form of adiabatic perturbation theory.

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